

Package ‘unbalhaar’

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for nonparametric function estimation in Gaussian noise using
Unbalanced Haar wavelets.

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unbalhaar-package

Function estimation via Unbalanced Haar wavelets

Description

The package implements top-down and bottom-up algorithms for nonparametric function estimation in Gaussian noise using Unbalanced Haar wavelets.

Details

Package:	unbalhaar
Type:	Package
Version:	2.0
Date:	2010-08-09
License:	GPL-2
LazyLoad:	yes

The main functions of the package are [uh](#) and [uh.bu](#).

Author(s)

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References

P. Fryzlewicz (2007) “Unbalanced Haar technique for nonparametric function estimation”. *Journal of the American Statistical Association*, 102, 1318-1327.

Examples

```
x <- c(rep(0, 100), rep(1, 200)) + rnorm(300)
est.topdown <- uh(x)
est.bottomup <- uh.bu(x)
```

best.unbal.haar	<i>Best top-down Unbalanced Haar decomposition</i>
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Description

The function finds the “best” top-down Unbalanced Haar (UH) decomposition of the input vector x , according to a selection rule (*criterion*) which specifies which UH vector gets chosen at each scale and location.

Usage

```
best.unbal.haar(x, criterion = inner.prod.max)
```

Arguments

x	a vector
<i>criterion</i>	a function which takes a vector of length n and returns an integer between 1 and $n-1$

Value

<i>tree</i>	A list of J matrices, where J represents the number of “scales”. Each matrix is of size $5 \times$ (the number of UH coefficients at a given scale). Each column (= vector of length 5) contains an Unbalanced Haar coefficient in the following format: 1st component - an index of the coefficient; 2nd component - the value of the coefficient; 3rd component - time point where the corresponding UH vector starts; 4th component - last time point before the breakpoint of the UH vector; 5th component - end point of the UH vector.
<i>smooth</i>	the “smooth” component of x , equal to $\text{sum}(x) / \sqrt{n}$, where n is the length of x

Author(s)

Piotr Fryzlewicz

See Also

[inner.prod.max](#), [inner.prod.max.p](#), [best.unbal.haar.bu](#)

Examples

```
best.unbal.haar(rnorm(100), inner.prod.max.p)
```

best.unbal.haar.bu	<i>Best bottom-up Unbalanced Haar decomposition</i>
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Description

The function finds the “best” bottom-up Unbalanced Haar (UH) decomposition of the input vector x .

Usage

```
best.unbal.haar.bu(x, stretch = length(x))
```

Arguments

<code>x</code>	a vector
<code>stretch</code>	at each iteration, only the first <code>1:stretch</code> elements of the current input vector (whose length decreases by one with each iteration) get scanned in the search for the worst-fitting fine-scale Unbalanced Haar wavelet

Value

<code>detail</code>	A matrix of size $3 \times n-1$, where n is the length of x , containing the detail coefficients of x in the order they were chosen. Each column corresponds to a single coefficient and contains, from top to bottom: location of the coefficient, the associated weight, and the value of the coefficient.
<code>smooth</code>	the “smooth” component of x , equal to $\text{sum}(x) / \sqrt{n}$, where n is the length of x

Author(s)

Piotr Fryzlewicz

See Also

[best.unbal.haar](#)

Examples

```
best.unbal.haar.bu(rnorm(100))
```

hard.thresh	<i>Hard thresholding of a top-down Unbalanced Haar decomposition</i>
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Description

Presented with an object returned by `best.unbal.haar`, the function sets to zero those Unbalanced Haar coefficients which fall below a certain threshold `sigma`.

Usage

```
hard.thresh(buh, sigma = 1)
```

Arguments

buh	an object returned by <code>best.unbal.haar</code> containing the decomposition to be thresholded
sigma	the threshold (a positive scalar)

Value

a thresholded object, of the same class as `buh`

Author(s)

Piotr Fryzlewicz

See Also

[best.unbal.haar](#), [hard.thresh.bu](#)

Examples

```
x <- rnorm(1000)
x.uh <- best.unbal.haar(x)
x.uh.th <- hard.thresh(x.uh)
x.uh.th.r <- reconstr(x.uh.th)
ts.plot(x.uh.th.r)
```

hard.thresh.bu	<i>Hard thresholding of a bottom-up Unbalanced Haar decomposition</i>
----------------	---

Description

Presented with an object returned by `best.unbal.haar.bu`, the function sets to zero those Unbalanced Haar coefficients which fall below a certain threshold `sigma`.

Usage

```
hard.thresh.bu(buh.bu, sigma = 1)
```

Arguments

<code>buh.bu</code>	an object returned by <code>best.unbal.haar.bu</code> containing the decomposition to be thresholded
<code>sigma</code>	the threshold (a positive scalar)

Value

a thresholded object, of the same class as `buh.bu`

Author(s)

Piotr Fryzlewicz

See Also

[best.unbal.haar.bu](#), [hard.thresh](#)

Examples

```
x <- rnorm(1000)
x.uh <- best.unbal.haar.bu(x)
x.uh.th <- hard.thresh.bu(x.uh)
x.uh.th.r <- reconstr.bu(x.uh.th)
ts.plot(x.uh.th.r)
```

inner.prod.iter*Inner products with Unbalanced Haar wavelets*

Description

For an input vector of length n , the function computes inner products between the input vector and all possible $n-1$ Unbalanced Haar vectors of length n .

Usage

```
inner.prod.iter(x)
```

Arguments

`x` a vector of length n

Details

The computation is iterative and is performed in computational time $O(n)$.

Value

a vector of length $n-1$, containing inner products between `x` and consecutive Unbalanced Haar wavelets of length n

Author(s)

Piotr Fryzlewicz

Examples

```
inner.prod.iter(rnorm(100))
```

inner.prod.max*Unbalanced Haar wavelet which maximises the inner product*

Description

The function finds the Unbalanced Haar vector which yields the largest (in absolute value) inner product with the input vector.

Usage

```
inner.prod.max(x)
```

Arguments

x a vector

Value

The index where `abs(inner.prod.iter(x))` is maximised. If two or more maxima are found, the med of their locations is returned.

Author(s)

Piotr Fryzlewicz

See Also

[inner.prod.iter](#), [med](#), [inner.prod.max.p](#)

Examples

```
inner.prod.max(c(rep(0, 100), rep(1, 200)))
```

inner.prod.max.p

Unbalanced Haar wavelet which maximises the inner product

Description

The function finds the Unbalanced Haar vector which yields the largest (in absolute value) inner product with the input vector, amongst those Unbalanced Haar vectors whose breakpoint is located between $100(1-p)\%$ and $100p\%$ of their support.

Usage

```
inner.prod.max.p(x, p = 0.8)
```

Arguments

x a vector
p a scalar in (0.5, 1]

Value

The index where `abs(inner.prod.iter(x))` is maximised on the subinterval $(1+\text{floor}((1-p)*n)):\text{ceiling}(p*n)$, where n is the length of x . If two or more maxima are found, the med of their locations is returned.

Author(s)

Piotr Fryzlewicz

See Also

[inner.prod.iter](#), [med](#), [inner.prod.max](#)

Examples

```
inner.prod.max.p(c(rep(0, 100), rep(1, 200)), .55)
```

med	<i>Median</i>
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Description

The function computes the median of a vector. Unlike `median`, it is guaranteed to return a value which is a component of the input vector.

Usage

```
med(x)
```

Arguments

x a vector

Value

a scalar defined as `quantile(x, .5, type=3)[[1]]`

Author(s)

Piotr Fryzlewicz

Examples

```
med(1:4)
median(1:4)
```

reconstr	<i>Reconstruct a top-down Unbalanced Haar decomposition</i>
----------	---

Description

Reconstructs a vector from its top-down Unbalanced Haar decomposition stored in an object returned by `best.unbal.haar` or `hard.thresh`.

Usage

```
reconstr(buh)
```

Arguments

`buh` an object of the type returned by `best.unbal.haar` and `hard.thresh`

Value

the inverse Unbalanced Haar transform of `buh`

Author(s)

Piotr Fryzlewicz

See Also

[best.unbal.haar](#), [hard.thresh](#), [reconstr.bu](#)

Examples

```
x <- rnorm(1000)
x.uh <- best.unbal.haar(x)
x.uh.th <- hard.thresh(x.uh)
x.uh.th.r <- reconstr(x.uh.th)
ts.plot(x.uh.th.r)
```

reconstr.bu	<i>Reconstruct a bottom-up Unbalanced Haar decomposition</i>
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Description

Reconstructs a vector from its bottom-up Unbalanced Haar decomposition stored in an object returned by `best.unbal.haar.bu` or `hard.thresh.bu`.

Usage

```
reconstr.bu(buh.bu)
```

Arguments

`buh.bu` an object of the type returned by `best.unbal.haar.bu` and `hard.thresh.bu`

Value

the inverse Unbalanced Haar transform of `buh.bu`

Author(s)

Piotr Fryzlewicz

See Also

[best.unbal.haar.bu](#), [hard.thresh.bu](#), [reconstr](#)

Examples

```
x <- rnorm(1000)
x.uh <- best.unbal.haar.bu(x)
x.uh.th <- hard.thresh.bu(x.uh)
x.uh.th.r <- reconstr.bu(x.uh.th)
ts.plot(x.uh.th.r)
```

uh

Denoising via top-down Unbalanced Haar

Description

Given an input vector of the form “signal + iid Gaussian noise”, the function estimates the noise level via Median Absolute Deviation, finds the best top-down Unbalanced Haar decomposition (according to the selection rule `criterion`), thresholds it with the universal threshold, and performs the inverse Unbalanced Haar transform to yield an estimate of the signal.

Usage

```
uh(x, criterion = inner.prod.max)
```

Arguments

`x` a vector of the form “signal + iid Gaussian noise”

`criterion` a function which takes a vector of length n and returns an integer between 1 and $n-1$

Value

an estimate of the signal

Author(s)

Piotr Fryzlewicz

References

P. Fryzlewicz (2007) “Unbalanced Haar technique for nonparametric function estimation”. *Journal of the American Statistical Association*, 102, 1318-1327.

See Also

[uh.bu](#), [best.unbal.haar](#), [inner.prod.max](#), [inner.prod.max.p](#), [hard.thresh](#), [reconstr](#)

Examples

```
x <- c(rep(0, 100), rep(1, 200)) + rnorm(300)
est <- uh(x)
```

uh.bu

*Denoising via bottom-up Unbalanced Haar***Description**

Given an input vector of the form “signal + iid Gaussian noise”, the function estimates the noise level via Median Absolute Deviation, finds the best bottom-up Unbalanced Haar decomposition, thresholds it with the universal threshold, and performs the inverse Unbalanced Haar transform to yield an estimate of the signal.

Usage

```
uh.bu(x, stretch = length(x))
```

Arguments

x	a vector of the form “signal + iid Gaussian noise”
stretch	at each iteration, only the first 1:stretch elements of the current input vector (whose length decreases by one with each iteration) get scanned in the search for the worst-fitting fine-scale Unbalanced Haar wavelet

Value

an estimate of the signal

Author(s)

Piotr Fryzlewicz

References

P. Fryzlewicz (2007) “Unbalanced Haar technique for nonparametric function estimation”. *Journal of the American Statistical Association*, 102, 1318-1327.

See Also

[uh](#), [best.unbal.haar.bu](#), [hard.thresh.bu](#), [reconstr.bu](#)

Examples

```
x <- c(rep(0, 100), rep(1, 200)) + rnorm(300)
est <- uh.bu(x)
```

unbal.haar.vector	<i>Unbalanced Haar vector</i>
-------------------	-------------------------------

Description

Computes the non-zero part of an Unbalanced Haar vector with a given start-, break- and end-point.

Usage

```
unbal.haar.vector(a)
```

Arguments

a	a three-component vector of integers such that $a[1] \leq a[2] < a[3]$. The three components specify, respectively, the start point, the time point just before the breakpoint, and the endpoint of the desired Unbalanced Haar vector.
---	--

Value

the non-zero part of the corresponding Unbalanced Haar vector

Author(s)

Piotr Fryzlewicz

Examples

```
unbal.haar.vector(c(1, 1, 2))
unbal.haar.vector(c(2, 5, 12))
```

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